a MIND for NUMBERS

HOW TO EXCEL AT MATH AND SCIENCE

(Even If You Flunked Algebra)

BARBARA OAKLEY, Ph.D.

a mind for numbers

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A Mind for Numbers is dedicated to Dr. Richard Felder, whose brilliance and passion have launched extraordinary improvements worldwide in the teaching of science, mathematics, engineering, and technology. My own successes, like those of tens of thousands of other educators, grow out of his fertile educational approaches. *Il miglior maestro.*

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The Law of Serendipity: Lady Luck favors the one who tries

contents

foreword

by Terrence J. Sejnowski,
Francis Crick Professor,
Salk Institute for Biological Studies

Your brain has amazing abilities, but it did not come with an instruction manual. You'll find that manual in *A Mind for Numbers*. Whether you're a novice or an expert, you will find great new ways to improve your skills and techniques for learning, especially related to math and science.

Henri Poincaré was a nineteenth-century mathematician who once described how he cracked a difficult mathematical problem that he had been intensively working on for weeks without success. He took a vacation. As he was getting on a bus in the south of France, the answer to the problem suddenly came to him, unbidden, from a part of his brain that had continued to work on the problem while he was enjoying his vacation. He knew he had the right solution even though he did not write down the details until he later returned to Paris.

What worked for Poincaré can work for you too, as Barbara Oakley explains in this insightful book. Surprisingly, your brain can also work on a problem even while you are sleeping and are not aware of anything. But it does this only if you concentrate on trying to solve the problem before falling asleep. In the morning, as often as not, a fresh insight will pop to mind that can help you solve the problem. The intense effort before a vacation or falling asleep is important for priming your brain; otherwise it will work on some other problem. There is nothing special about math or science in this regard—your brain will work just as hard solving social problems as on math and science problems, if that is what has been on your mind recently.

You will find many more insights and techniques about how to learn effectively in this fascinating and timely book, which looks at learning as an adventure rather than hard labor. You will see how you can fool yourself about whether you actually know the material; you will find ways to hold your focus and space out your practice; and you will learn to condense key ideas so you can hold them more easily in your mind. Master the simple, practical approaches outlined here and you will be able to learn more effectively and with less frustration. This wonderful guide will enrich both your learning and your life.

—Terrence J. Sejnowski, Francis Crick Professor,
Salk Institute for Biological Studies

preface

by Jeffrey D. Karpicke,
James V. Bradley Associate Professor of
Psychological Sciences at Purdue University

This book can make a profound difference in how you look at and understand learning. You will learn the *simplest*, *most effective*, and *most efficient* techniques researchers know about how to learn. And you'll have fun while you're doing it.

What's surprising is that a lot of learners use ineffective and inefficient strategies. In my laboratory, for example, we have surveyed college students about their learning. They most commonly use the strategy of *repeated reading*—simply reading through books or notes over and over. We and other researchers have found that this passive and shallow strategy often produces minimal or no learning. We call this "labor in vain"—students are putting in labor but not getting anywhere.

We don't engage in passive rereading because we are dumb or lazy. We do it because we fall prey to a cognitive illusion. When we read material over and over, the material becomes familiar and fluent, meaning it is easy for our minds to process. We then think that this easy processing is a sign that we have learned something well, even though we have not.

This book will introduce you to this and other illusions of learning and give you tools to overcome them. And it will introduce great new tools, such as retrieval practice, that can have a powerful effect in boosting the "bang for your buck" from your time spent in learning. It's a deeply practical yet inspiring book that helps you clearly see why some approaches are so much more effective than others.

We're on the edge of an explosion in knowledge about how to learn most effectively. In this new world of insight, you'll find *A Mind for Numbers* to be an indispensable guide.

—Jeffrey D. Karpicke, James V. Bradley, Associate Professor of Psychological Sciences at Purdue University

To the Reader

People who work professionally with math and science often spend years discovering effective learning techniques. Once they've figured these methods out, *great*! They have unwittingly passed the initiation rites needed to join the mysterious society of math and science practitioners.

I've written this book to lay out these simple techniques so that you can immediately begin using them. What takes years for practitioners to discover is now at your fingertips.

Using these approaches, no matter what your skill levels in math and science, you can change your thinking and change your life. If you're already an expert, this peek under the mental hood will give you ideas for turbocharging successful learning, including counter-intuitive test-taking tips and insights that will help you make the best use of your time on homework and problem sets. If you're struggling, you'll see a structured treasure trove of practical techniques that walk you through what you need to do to get on track. If you've ever wanted to become better at anything, this book will help serve as your guide.

This book is for high school students who love art and English classes but loathe math. It is meant as well as for college students who already excel in math, science, engineering, and business, but who suspect there are mental tools to be added to their learning toolkits. It's for parents whose children are either falling off the math track or trying to rocket toward math and science stardom. It's for the frazzled nine-to-five worker who hasn't been able to pass an important certification test, and for the night-shift convenience store clerk who has dreamed of becoming a nurse—or even a doctor. It's for the growing army of homeschoolers. It's for teachers and professors, not only in math, science, engineering, and technology, but also in fields such as education, psychology, and business. It's for the retiree who finally has the time to embrace new knowledge in computing, for example, or the intricacies of great cooking. And it's for readers of all ages who love to learn a little about everything.

In short, this book is for you. Enjoy!

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open the door

What are the odds you'd open your refrigerator door and find a zombie in there, knitting socks? The odds are about the same that a touchy-feely, language-oriented person like me would end up as a professor of engineering.

Growing up, I *hated* math and science. I flunked my way through high school math and science courses, and only started studying trigonometry—remedial trigonometry—when I was twenty-six years old.

As a youngster, even the simple concept of reading a clock face didn't seem to make sense to me. Why should the little hand point toward the hour? Shouldn't it be the big hand, since the hour was more important than the minute? Did the clock read ten ten? Or one fifty? I was perpetually confused. Worse than my problems with clocks was the television. In those days before the remote control, I didn't even know which button turned the television on. I watched a show only in the company of my brother or sister. They not only could turn the TV on, but could also tune the channel to the program we wanted to watch. Nice.

All I could conclude, looking at my technical ineptitude and flunking grades in math and science, was that I wasn't very smart. At least, not that way. I didn't realize it then, but my self-portrait as being technically, scientifically, and mathematically incapable was shaping my life. At the root of it all was my problem with mathematics. I had come to think of numbers and equations as akin to one of life's deadly diseases—to be avoided at all costs. I didn't realize then that there were simple mental tricks that could have brought math into focus for me, tricks that are helpful not only for people who are bad at math, but also for those who are already good at it. I didn't understand that my type of thinking is typical of people who believe they can't do math and science. Now, I realize that my problem was rooted in two distinctly different modes for viewing the world. Back then, I only knew how to tap one mode for learning—and the result was that I was deaf to the music of math.

Mathematics, as it's generally taught in American school systems, can be a saintly mother of a subject. It climbs logically and majestically from addition through subtraction, multiplication, and division. Then it sweeps up toward the heavens of mathematical beauty. But math can also be a wicked stepmother. She is utterly unforgiving if you happen to miss any step of the logical sequence—and missing a step is easy to do. All you need is a disruptive family life, a burned-out teacher, or an unlucky extended bout with illness—even a week or two at a critical time can throw you off your game.

Or, as was the case with me, simply no interest or seeming talent whatsoever.

In seventh grade, disaster struck my family. My father lost his job after a serious back injury. We ended up in a hardscrabble school district where a crotchety math teacher made us sit for hours in the sweltering heat doing rote addition and multiplication. It didn't help that Mr. Crotchety refused to provide any explanations. He seemed to enjoy seeing us flounder.

By this time, I not only didn't see any use for math—I actively loathed it. And as far as the sciences went—well, they didn't. In my first chemistry experiment, my teacher chose to give my lab partner

Me at age ten with Earl the lamb. I loved critters, reading, and dreaming. Math and science weren't on my play list.

and me a different substance than the rest of the class. He ridiculed us when we fudged the data in an attempt to match everyone else's results. When my well-meaning parents saw my failing grades and urged me to get help during the teacher's office hours, I felt I knew better. Math and science were worthless, anyway. The Gods of Required Coursework were determined to shove math and science down my throat. My way of winning was to refuse to understand anything that was taught, and to belligerently flunk every test. There was no way to outmaneuver my strategy.

I did have other interests, though. I liked history, social studies, culture, and especially language. Luckily, those subjects kept my grades afloat.

Right out of high school, I enlisted in the army because they would actually pay me to learn another language. I did so well in studying Russian (a language I'd selected on a whim) that an ROTC scholarship came my way. I headed off to the University of Washington to get a bachelor's degree in Slavic languages and literature, where I graduated with honors. Russian flowed like warm syrup—my accent was so good that I found myself mistakenly taken on occasion for a native speaker. I spent lots of time gaining this expertise—the better I got, the more I enjoyed what I was doing. And the more I enjoyed what I was doing, the more time I spent on it. My success reinforced my desire to practice, and that built more success.

But in the most unlikely situation I could have ever imagined, I eventually found myself commissioned as a second lieutenant in the U.S. Army Signal Corps. I was suddenly expected to become an expert in radio, cable, and telephone switching systems. What a turning point! I went from being on top of the world, an expert linguist, in control of my destiny, to being thrown into a new technological world where I was as stunted as a stump.

Yikes!

I was made to enroll in mathematically oriented electronics training (I finished at the bottom of the class), and then off I went to West Germany, where I became a pitiable communications platoon leader. I saw that the officers and enlisted members who *were* technically competent were in demand. They were problem solvers of the first order, and their work helped everyone accomplish the mission.

I reflected on the progress of my career and realized that I'd followed my inner passions without also being open to developing new ones. As a consequence, I'd inadvertently pigeonholed myself. If I stayed in the army, my poor technical know-how would always leave me a second-class citizen.

On the other hand, if I left the service, what could I do with a degree in Slavic languages and literature? There aren't a lot of jobs for Russian linguists. Basically, I'd be competing for entry-level secretarial-type jobs with millions of others who also had bachelor's of arts degrees. A purist might argue that I'd distinguished myself in both my studies and my service and could find much better work, but that purist would be unaware of how tough the job market can sometimes be.

Fortunately there was another unusual option. One of the great benefits of my service was that I had GI Bill money to offset the costs of future schooling. What if I used that support to do the unthinkable and try to retrain myself? Could I retool my brain from math-phobe to math lover? From technophobe to technogeek?

I'd never heard of anyone doing anything like that before, and certainly not coming from the phobic depths I'd sunk to. There couldn't possibly be anything more foreign to my personality than mastering math and science. But my colleagues in the service had shown me the concrete benefits of doing so.

It became a challenge—an irresistible challenge.

I decided to retrain my brain.

It wasn't easy. The first semesters were filled with frightening frustration. I felt like I was wearing a blindfold. The younger students around me mostly seemed to have a natural knack for seeing the solutions, while I was stumbling into walls.

But I began to catch on. Part of my original problem, I found, was that I had been putting my effort forth in the wrong way—like trying to lift a piece of lumber when you're standing on it. I began to pick up little tricks about not only how to study but when to quit. I learned that internalizing certain concepts and techniques could be a powerful tool. I also learned not to take on too much at once, allowing myself plenty of time to practice even if it meant my classmates would sometimes graduate ahead of me because I wasn't taking as many courses each semester as they were.

As I gradually *learned how to learn* math and science, things became easier. Surprisingly, just as with studying language, the better I got, the more I enjoyed what I was doing. This former Queen of the Confused in math went on to earn a bachelor's degree in electrical engineering and then a master's in electrical and computer engineering. Finally, I earned a doctorate in systems engineering, with a broad background that included thermodynamics, electromagnetics, acoustics, and physical chemistry. The higher I went, the better I did. By the time I reached my doctoral studies, I was breezing by with perfect grades. (Well, perhaps not quite breezing. Good grades still took work. But the work I needed to do was clear.)

Now as a professor, I have become interested in the inner workings of the brain. My interest grew naturally from the fact that engineering lies at the heart of the medical images that allow us to tease out how the brain functions. I can now more clearly see how and why I was able to change my brain. I also see how I can help you learn more effectively without the frustration and struggle I experienced.[1] And as a researcher whose work straddles engineering, the social sciences, and the humanities, I'm also aware of the essential creativity underlying not just art and literature, but also math and science.

If you don't (yet) consider yourself naturally good at math and science, you may be surprised to learn that **the brain is *designed* to**

do extraordinary mental calculations. We do them every time we catch a ball, or rock our body to the beat of a song, or maneuver our car around a pothole in the road. We often do complex calculations, solving complex equations unconsciously, unaware that we sometimes already know the solution as we slowly work toward it.[2] In fact, we all have a natural feel and flair for math and science. Basically, we just need to master the lingo and culture.

In writing this book, I connected with hundreds of the world's leading professor-teachers of mathematics, physics, chemistry, biology, and engineering, as well as education, psychology, and professional disciplines such as business and the health sciences. It was startling to hear how often these world-class experts had used precisely the approaches outlined in the book when they themselves were learning their disciplines. These techniques were also what the experts asked their students to use—but since the methods sometimes seem counterintuitive, and even irrational, instructors have often found it hard to convey their simple essence. In fact, because some of these learning and teaching methods are derided by ordinary instructors, superstar teachers sometimes divulged their teaching and learning secrets to me with embarrassment, unaware that many other top instructors shared similar approaches. By collecting many of these rewarding insights in one place, you too can easily learn and apply practical techniques gleaned in part from these "best of the best" teachers and professors. These techniques are especially valuable for helping you learn more deeply and effectively in limited time frames. You'll also gain insight from students and other fellow learners—people who share your constraints and considerations.

Remember, this is a book for math experts and mathphobes alike. This book was written to make it easier for you to learn math and science, regardless of your past grades in those subjects or how good or bad you think you are at them. It is designed to expose your thought processes so you can understand how your mind learns—and also how your mind sometimes fools you into believing you're learning, when you're actually not. The book also includes plenty of skill-building exercises that you can apply directly to your current studies. **If you're *already* good at numbers or science, the insights in this book can help make you better.** They will broaden your enjoyment, creativity, and equation-solving elegance.

If you're simply convinced you don't have a knack for numbers or science, this book may change your mind. You may find it hard to believe, but there's hope. When you follow these concrete tips based on how we actually learn, you'll be amazed to see the changes within yourself, changes that can allow new passions to bloom.

What you discover will help you be more effective and creative, not only in math and science, but in almost everything you do.

Let's begin!

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easy does it:

Why Trying Too Hard Can Sometimes Be Part of the Problem

If you want to understand some of the most important secrets to learning math and science, look at this picture.

The man on the right is legendary chess grand master Garry Kasparov. The boy on the left is thirteen-year-old Magnus Carlsen. Carlsen has just wandered away from the board during the height of a speed chess game, where little time is given to think about moves or strategy. That's a little like casually deciding to do a backflip while walking a tightrope across Niagara Falls.

Yes, Carlsen was psyching out his opponent. Rather than obliterating the upstart youngster, the flustered Kasparov played to a draw. But the brilliant Carlsen, who would go on to become the youngest top-rated chess player in history, was doing something far beyond playing mind games with his older opponent. Gaining insight into Carlsen's approach can help us understand how the mind

Thirteen-year-old Magnus Carlsen (left), and legendary genius Garry Kasparov playing speed chess at the "Reykjavik Rapid" in 2004. Kasparov's shock is just beginning to become apparent.

learns math and science. Before we go into how Carlsen psyched out Kasparov, we need to cover a couple of important ideas about how people think. (But I promise, we'll come back to Carlsen.)

We're going to be touching on some of the main themes of the book in this chapter, so don't be surprised if you have to toggle around a bit in your thinking. Being able to toggle your thinking—getting a glimpse of what you are learning before returning later to more fully understand what's going on, is itself one of the main ideas in the book!

NOW YOU TRY!

Prime Your Mental Pump

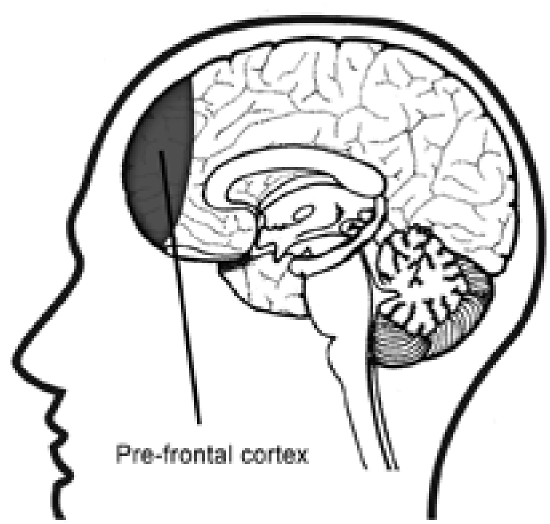
As you first begin looking at a chapter or section of a book that teaches concepts of math or science, it helps to take a "picture walk" through the chapter, glancing not only at the graphics, diagrams, and photos, but also at the section headings, summary, and even questions at the end of the chapter, if the book has them. This seems counterintuitive—you haven't actually read the chapter yet, but it helps prime your mental pump. So go ahead now and glance through this chapter.

You'll be surprised at how *spending a minute or two glancing ahead before you read in depth will help you organize your thoughts*. You're creating little neural hooks to hang your thinking on, making it easier to grasp the concepts.

Focused versus Diffuse Thinking

Since the very beginning of the twenty first century, neuroscientists have been making profound advances in understanding the two different networks that the brain switches between—a *highly attentive state* and a more relaxed *default mode network*.[1] We'll call the thinking processes related to these networks the **focused mode** and **diffuse mode**, respectively—these modes are highly important for learning.[2] It seems you frequently switch back and forth between these two modes in your day-to-day activities. You're in either one mode or the other—not consciously in both at the same time. The diffuse mode does seem to be able to work quietly in the background on something you are not actively focusing on.[3]

Focused-mode thinking is essential for studying math and science. It involves a direct approach to solving problems using rational, sequential, analytical approaches. The focused mode is associated with the concentrating abilities of the brain's prefrontal cortex, located right behind your forehead.[4] Turn your attention to something and *bam*—the focused mode is *on*, like the tight, penetrating beam of a flashlight.



The prefrontal cortex is the area right behind the forehead.

Diffuse-mode thinking is also essential for learning math and science. It allows us to suddenly gain a new insight on a problem we've been struggling with and is associated with "big picture" perspectives. Diffuse-mode thinking is what happens when you relax your attention and just let your mind wander. This relaxation can allow different areas of the brain to hook up and return valuable insights. Unlike the focused mode, the diffuse mode doesn't seem to be affiliated with any one area of the brain—you can think of it as being "diffused" throughout the brain.[5] Diffuse-mode insights

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often flow from preliminary thinking that's been done in the focused mode. (The diffuse mode must have clay to make bricks!)

Learning involves a complex flickering of neural processing among different areas of the brain, as well as back and forth between hemispheres.[6] So this means that thinking and learning is more complicated than simply switching between the focused and diffuse modes. But fortunately, we don't need to go deeper into the physical mechanisms. We're going to take a different approach.

The Focused Mode—A Tight Pinball Machine

To understand focused and diffuse mental processes, we're going to play some pinball. (Metaphors are powerful tools for learning in math and science.) In the old game of pinball, you pull back on a spring-loaded plunger and it whacks a ball, which ends up bouncing randomly around the circular rubber bumpers.

Look at the following illustration. When you focus your attention on a problem, your mind pulls back the mental plunger and releases a thought. Boom—that thought takes off, bumping around like the pinball in the head on the left. This is the *focused mode* of thinking.

Notice how the round bumpers are very close together in the focused mode. In contrast, the diffuse mode on the right has its circular rubber bumpers farther apart. (If you want to pursue the metaphor still further, you can think of each bumper as a cluster of neurons.)

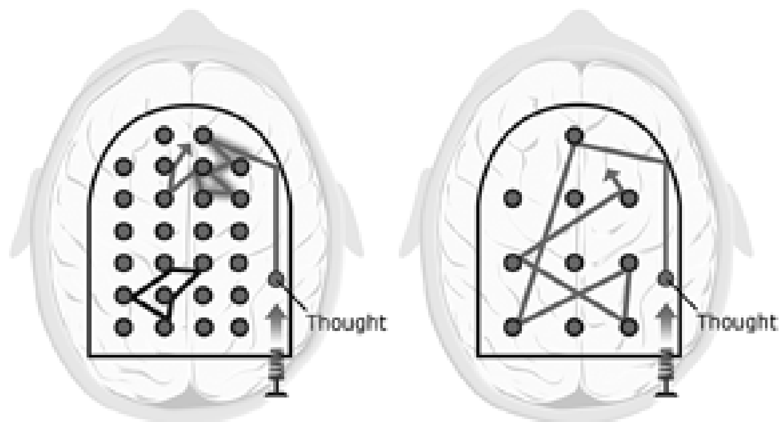
The close bumpers of the focused mode mean that you can more easily think a precise thought. Basically, the focused mode is used to concentrate on something that's already tightly connected in your mind, often because you are familiar and comfortable with

FPO

This happy zombie is playing neural pinball.

the underlying concepts. If you look closely at the upper part of the focused-mode thought pattern, you'll see a wider, "well-trodden" part of the line. That broader path shows how the focused-mode thought is following along a route you've already practiced or experienced.

For example, you can use the focused mode to multiply numbers—if you already know how to multiply, that is. If you're studying a language, you might use the focused mode to become



In the game "pinball," a ball, which represents a thought, shoots up from the spring-loaded plunger to bounce randomly against rows of rubber bumpers. These two pinball machines represent focused (left) and diffuse (right) ways of thinking. The focused approach relates to intense concentration on a specific problem or concept. But sometimes you can inadvertently find yourself focusing intently and trying to solve a problem using erroneous thoughts that are in a different place in the brain from the "solution" thoughts you need to actually need to solve the problem.

As an example of this, note the upper "thought" that your pinball first bounces around in on the left-hand image. It is very far away and completely unconnected from the lower pattern of thought in the same brain. You can see how part of the upper thought seems to have an underlying broad path. This is because you've thought something similar to that thought before. The lower thought is a new thought—it doesn't have that underlying broad pattern.

The diffuse approach on the right involves a big-picture perspective. This thinking mode is useful when you are learning something new. As you can see, the diffuse mode doesn't allow you to focus tightly and intently to solve a specific problem—but it can allow you to get closer to where that solution lies because you're able to travel much farther before running into another bumper.

more fluent with the Spanish verb conjugation you learned last week. If you're a swimmer, you might use the focused mode to analyze your breast stroke as you practice staying low to allow more energy to go into your forward motion.

When you focus on something, the consciously attentive prefrontal cortex automatically sends out signals along neural pathways. These signals link different areas of your brain related to what you're thinking about. This process is a little like an octopus that sends its tentacles to different areas of its surroundings to fiddle with whatever it's working on. The octopus has only so many tentacles to make connections, just as your working memory has only so many things it can hold at once. (We'll talk more about the working memory later.)

You often first funnel a problem into your brain by focusing your attention on words—reading the book or looking at your notes from lecture. Your attentional octopus activates your focused mode. As you do your initial focused noodling around with the problem, you are thinking tightly, using the pinball bumpers that are close together to follow along familiar neural pathways. This isn't a concern if you are trying to figure out something very similar to what you already know. Your thoughts rattle easily through the previously ingrained patterns and quickly settle on a solution. In math and science, however, it often doesn't take much of a change for a problem to become quite different. Problem-solving then grows more difficult.

Why Math and Science Can Be More Challenging

Focused problem solving in math and science is often more effortful than focused-mode thinking involving language and people.[7] This may be because humans haven't evolved over the millennia to manipulate mathematical ideas, which are frequently more abstractly encrypted than those of conventional language.[8] Obviously, we can still think *about* math and science—it's just that the *abstractness* and *encryptedness* adds a level—sometimes a number of levels—of complexity.

What do I mean by abstractness? You can point to a real live *cow* chewing its cud in a pasture and equate it with the letters *c-o-w* on the page. But you can't point to a real live *plus sign* that the symbol "+" is modeled after—the idea underlying the plus sign is more *abstract*. By *encryptedness*, I mean that one symbol can stand for a number of different operations or ideas, just as the multiplication sign symbolizes repeated addition. In our pinball analogy, it's as if the abstractness and encryptedness of math can make the pinball bumpers a bit spongier—it takes extra practice for the bumpers to harden and the pinball to bounce properly. This is why dealing with procrastination, while important in studying any discipline, is particularly important in math and science. We'll be talking more about this later.

Related to these difficulties in math and science is another challenge. It's called the *Einstellung* **effect** (pronounced *EYE-nshtellung*). In this phenomenon, an idea you already have in mind, or your simple initial thought, prevents a better idea or solution from being found.[9] We saw this in the focused pinball picture, where your initial pinball thought went to the upper part of the brain, but the solution thought pattern was in the lower part of the image. (The German word *Einstellung* means "installation"—basically you can remember *Einstellung* as *installing* a roadblock because of the way you are initially looking at something.)

This kind of wrong approach is especially easy to do in science because sometimes your initial intuition about what's happening is misleading. You have to unlearn your erroneous older ideas even while you're learning new ones.[10]

The *Einstellung* effect is a frequent stumbling block for students. It's not just that sometimes your natural intuitions need to be retrained—it's that sometimes it is tough even figuring out where to begin, as when tackling a homework problem. You bumble about—your thoughts far from the actual solution—because the crowded bumpers of the focused mode prevent you from springing to a new place where the solution might be found.

This is precisely why **one significant mistake students sometimes make in learning math and science is** *jumping into the water before they learn to swim***.**[11] In other words, they blindly start working on homework without reading the textbook, attending lectures, viewing online lessons, or speaking with someone knowledgeable. This is a recipe for sinking. It's like randomly allowing a thought to pop off in the focused-mode pinball machine without paying any real attention to where the solution truly lies.

Understanding how to obtain *real* solutions is important, not only in math and science problem solving, but for life in general. For example, a little research, self-awareness, and even self-experimentation can prevent you from being parted with your money—or even your good health—on products that come with bogus "scientific" claims.[12] And just having a little knowledge of the relevant math can help prevent you from defaulting on your mortgage—a situation that can have a major negative impact on your life.[13]

The Diffuse Mode—A Spread-Out Pinball Machine

Think back several pages to the illustration of the diffuse-mode pinball machine brain, where the bumpers were spread far apart. This mode of thinking allows the brain to look at the world from a much broader perspective. Can you see how a thought can travel much further before it runs into a bumper? The connections are further apart—you can quickly zoom from one clump of thought to another that's quite far away. Of course, it's hard to think precise, complex thoughts while in this mode.

If you are grappling with a new concept or trying to solve a new problem, you don't have preexisting neural patterns to help guide your thoughts—there's no fuzzy underlying pathway to help guide you. You may need to range widely to encounter a potential solution. The diffuse mode is just the ticket!

Another way to think of the difference between focused and diffuse modes is to think of a flashlight. You can set a flashlight so it has a tight focused beam that can penetrate deeply into a small area. Or you can set the flashlight onto a more diffuse setting where it casts its light broadly, but not very strongly in any one area.

If you are trying to understand or figure out something new, your best bet is to turn off your precision-focused thinking and turn on your "big picture" diffuse mode, long enough to be able to latch onto a new, more fruitful approach. As we'll see, the diffuse mode has a mind of its own—you can't simply command it to turn on. But we'll soon get to some tricks that can help you transition between modes.

COUNTERINTUITIVE CREATIVITY

"When I was learning about the diffuse mode, I began to notice it in my daily life. For instance, I realized my best guitar riffs always came to me when I was 'just messing around' as opposed to when I sat down intent on creating a musical masterpiece (in which case my songs were often clichéd and uninspiring). Similar things happened when I was writing a school paper, trying to come up with an idea for a school project, or trying to solve a difficult math problem. I now follow the rule of thumb that is basically: The harder you push your brain to come up with something creative, the less creative your ideas will be. So far, I have not found a single situation where this does not apply. Ultimately, this means that relaxation is an important part of hard work—and good work, for that matter."

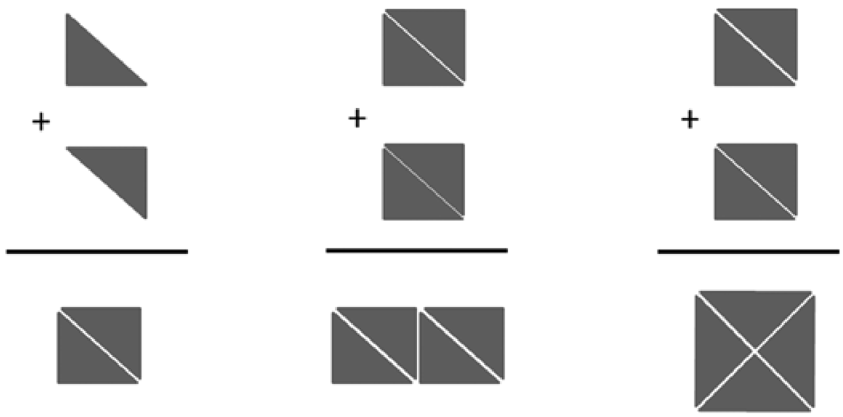
—Shaun Wassell, freshman, computer engineering

Why Are There Two Modes of Thinking?

Why do we have these two different thinking modes? The answer may be related to two major problems that vertebrates have had in staying alive and passing their genes on to their offspring. A bird, for example, needs to focus carefully so it can pick up tiny pieces of grain as it pecks the ground for food, and at the same time, it must scan the horizon for predators like hawks. What's the best way to carry out those two very different tasks? Split things up, of course. You can have one hemisphere of the brain more oriented toward the focused attention needed to peck at food and the other oriented toward scanning the horizon for danger. When each hemisphere tends toward a particular type of perception, it may increase the chance of survival.[14] If you watch birds, they'll first peck, and then pause to scan the horizon—almost as if they are alternating between focused and diffuse modes.

In humans, we see a similar splitting of brain functions. The left side of the brain is somewhat more associated with careful, focused attention. It also seems more specialized for handling sequential information and logical thinking—the first step leads to the second step, and so on. The right seems more tied to diffuse scanning of the environment and interacting with other people, and seems more associated with processing emotions.[15] It also is linked with handling simultaneous, big-picture processing.[16]

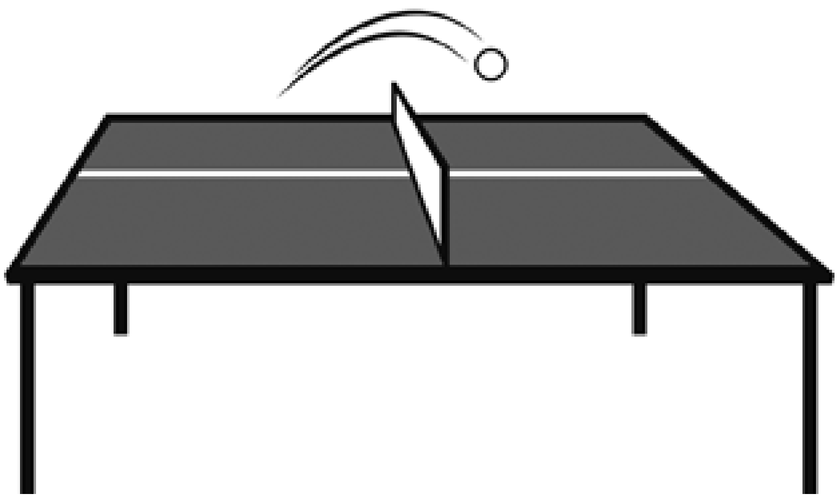
The slight differences in the hemispheres give us a sense of why two different processing modes may have arisen. But be wary of the idea that some people are "left-brain" or "right-brain" dominant—research indicates that is simply not true.[17] Instead it is clear that *both* hemispheres are involved in focused as well as diffuse modes of thinking. **To learn about and be creative in math and science,**



Here's a quick example that gives a sense of the difference between focused and diffuse thinking. If you are given two triangles to put together into a square shape, it's easy to do, as shown on the left. If you are given two more triangles and told to form a square, your first tendency is to erroneously put them together to form a rectangle, as shown in the middle. This is because you've already laid down a focused mode pattern that you have a tendency to follow. It takes an intuitive, diffuse leap to realize that you need to completely rearrange the pieces if you want to form another square, as shown on the right.[19]

we need to strengthen and use both the focused and diffuse modes.[18]

Evidence suggests that to grapple with a difficult problem, we must first put hard, focused-mode effort into it. (We learned that in grade school!) Here's the interesting part: The diffuse mode is *also* often an important part of problem solving, especially when the problem is difficult. *But as long as we are consciously focusing on a problem, we are blocking the diffuse mode.*



There's a winner at Ping-Pong only if the ball is able to go back and forth.

FPO

EMBRACE BEFUDDLEMENT!

"Befuddlement is a healthy part of the learning process. When students approach a problem and don't know how to do it, they'll often decide they're no good at the subject. Brighter students, in particular, can have difficulty in this way—their breezing through high school leaves them no reason to think that being confused is normal and necessary. But the learning process is all about working your way out of confusion. Articulating your question is 80 percent of the battle. By the time you've figured out what's confusing, you're likely to have answered the question yourself!"

—Kenneth R. Leopold, distinguished teaching professor, Department of Chemistry, University of Minnesota

The bottom line is that problem solving in any discipline often involves an exchange between the two fundamentally different modes. One mode will process the information it receives and then send the result back to the other mode. This volleying of information back and forth as the brain works its way toward a conscious solution appears essential for understanding and solving all but trivial problems and concepts.[20] The ideas presented here are extremely helpful for understanding learning in math and science. But as you are probably beginning to see, they can be just as helpful for many other subject areas, such as language, music, and creative writing.

NOW YOU TRY!

Shifting Modes

Here's a cognitive exercise that can help you feel the shift from focused to diffuse mode. See whether you can form a new triangle that points down by moving only three coins.

FPO

When you relax your mind, releasing your attention and focusing on nothing in particular, the solution can most easily come to you.

You should know that some children get this exercise instantly, while some highly intelligent professors finally just give up. To answer this question, it helps to summon your inner child. The solutions for this challenge and for all the "Now You Try!" challenges in the book can be found in the endnotes.[21]

Procrastination Prelude

Many people struggle with procrastination. We'll have a lot to say later in this book about how to deal effectively with procrastination. For now, keep in mind that **when you procrastinate, you are leaving yourself only enough time to do superficial focused-mode learning.** You are also increasing your stress level because you know you have to complete what feels like an unpleasant task. The resulting neural patterns will be faint and fragmented and will quickly disappear—you'll be left with a shaky foundation. In math and science in particular, this can create severe problems. If you cram for a test at the last minute or quickly breeze through your homework, you won't have time for either learning mode to help you tackle the tougher concepts and problems or to help you synthesize the connections in what you are learning.

NOW YOU TRY!

Focusing Intently but Briefly

If you often find yourself procrastinating, as many of us do, here's a tip. Turn off your phone and any sounds or sights (or websites) that might signal an interruption. Then set a timer for twenty-five minutes and put yourself toward doing a twenty-five-minute interlude of work focused on a task—any task. Don't worry about finishing the task—just worry about working on it. Once the twenty-five minutes is up, reward yourself with web surfing, checking your phone, or whatever you like to do. *This reward is as important as the work itself.* You'll be amazed at how productive a focused twenty-five-minute stint can be—especially when you're just focusing on the work itself, *not* on finishing. (This method, known as the Pomodoro technique, will be discussed in more detail in chapter 6.)

If you want to apply a more advanced version of this approach, imagine that at the end of the day, you are reflecting on the *one* most important task that you accomplished that day. What would that task be? Write it down. Then work on it. Try to complete at least three of these twenty-five-minute sessions that day, on whatever task or tasks you think are most important.

At the end of your workday, look at what you crossed off your list and savor the feeling of accomplishment. Then write a few key things that you would like to work on the next day. This early preparation will help your diffuse mode begin to think about how you will get those tasks done the next day.

SUMMING IT UP

- Our brain uses two very different processes for thinking—the focused and diffuse modes. You toggle back and forth between these modes, using one or the other.
- It is typical to be stumped by new concepts and problems when we first focus on them.
- To figure out new ideas and solve problems, it's important not only to focus initially, but also to subsequently turn our focus *away* from what we want to learn.
- The *Einstellung* effect refers to getting stuck in solving a problem or understanding a concept as a result of becoming fixated on a flawed approach. Switching modes from focused to diffuse can help free you from this effect. Keep in mind, then, that sometimes you will need to be flexible in your thinking. You may need to switch modes to solve a problem or understand a concept.

PAUSE AND RECALL

Close the book and look away. What were the main ideas of this chapter? Don't worry if you can't recall very much when you first begin trying this. As you continue practicing this technique, you'll begin noticing changes in how you read and how much you recall.

TO ENHANCE YOUR LEARNING

1. How would you recognize when you are in the diffuse mode? How does it feel to be in the diffuse mode?

2. When you are consciously thinking of a problem, which mode is active and which is blocked? What can you do to escape this blocking?

3. Recall an episode where you experienced the *Einstellung* effect. How were you able to change your thinking to get past the preconceived, but erroneous, notion?

4. Explain how the focused and diffuse modes might be equated to an adjustable beam on a flashlight. When can you see farther? When can you see more broadly, but less far?

5. Why is procrastination sometimes a special challenge for those who are studying math and science?

SHIFTING OUT OF BEING STUCK: INSIGHT FROM NADIA NOUI-MEHIDI, A SENIOR STUDYING ECONOMICS

"I took Calculus I in eleventh grade and it was a nightmare. It was so profoundly different from anything I had learned before that I didn't even know how to learn it. I studied longer and harder than I ever had before, yet no matter how many problems I did or how long I stayed in the library I was learning nothing. I ultimately just stuck to what I could get by with through memorizing. Needless to say, I did not do well on the AP (advanced placement) exam.

"I avoided math for the next two years, and then as a sophomore in college, I took Calculus I and got a 4.0. I don't think I was any smarter two years later, but there was a complete shift in the way I was approaching the subject.

"I think in high school I was stuck in the focused mode of thinking (*Einstellung!*) and felt that if I kept trying to approach problems in the same way it would eventually click.

"I now tutor students in math and economics and the issues are almost always that they are fixated on looking at the details of the problem for clues on how to solve it, and not on understanding the problem itself. I don't think you can tutor someone on how to think—it's kind of a personal journey. But here are some things that have helped me understand a concept that at first seems complicated or confusing.

1. I understand better when I read the book rather than listen to someone speak, so I always read the book. I skim first so I know basically what the chapter is trying to get at and then I read it in detail. I read the chapter more than once (but not in a row).

2. If after reading the book, I still don't fully understand what's going on, I Google or look at YouTube videos on the subject. This isn't because the book or professor isn't thorough, but

rather because sometimes hearing a slightly different way of phrasing something can make your mind look at the problem from a different angle and spark understanding.

3. I think the most clearly when I'm driving. Sometimes I'll just take a break and drive around—this helps a lot. I have to be somewhat occupied because if I just sit down and think I end up getting bored or distracted and can't concentrate.